This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2010 proceedings

The Optimal Transmission Power per Round for Hybrid-ARQ Rayleigh Fading Links

Weifeng Su, Sangkook Lee, and Dimitris A. Pados
Dept. of Electrical Engineering, SUNY at Buffalo, NY 14260, E-mails: {weifeng, sklee4, pados}@buffalo.edu
John D. Matyjas
Air Force Research Laboratory/RIGF, Rome, NY 13441, E-mail: John.Matyjas@rl.af.mil

***Abstract*— We address the fundamental problem of identifying the optimal power allocation sequence for hybrid automatic-repeat-request (H-ARQ) communications over quasi-static Rayleigh fading channels. For any targeted H-ARQ link outage probability, we find the sequence of power values that minimizes the average total expended transmission power. The newly founded power allocation solution reveals that conventional equal-power H-ARQ assignment is far from optimal. For example, for targeted outage probability of 10^{-3} with a maximum of two transmissions, the average total transmission power with optimal assignment is 9dB lower than the equal-power protocol. The difference in average total power cost grows further when the number of allowable retransmissions increases (for example, 11dB gain with a cap of 5 transmissions) or the targeted outage probability decreases (27dB gain with outage probability 10^{-5} and transmissions capped at 5).**

Index Terms:[1] **Hybrid automatic-repeat-request protocol, optimum power allocation, outage probability, Rayleigh fading.**

I. Introduction

Automatic-repeat-request (ARQ) protocols, in which a receiver requests retransmission when a packet is not correctly received, are commonly used in data link control to enable reliable data packet transmissions [1]–[7]. In a basic/simplest ARQ protocol, a receiver decodes an information packet based only on the received signal in each transmission round [1], [2]. Advanced ARQ schemes, in which a receiver may decode an information packet by combining received signals from all previous transmission rounds, have been known as hybrid ARQ (H-ARQ) protocols [3]–[7].

In wireless links formed by wireless devices with limited power resources, power efficiency is a key research matter in the optimization of ARQ retransmission protocols [8]–[12]. In [8], [9], the power efficiency of several ARQ protocols was examined under the assumption of the same transmission power in each retransmission round. In [10], the transmission power in each retransmission round was optimized for several ARQ protocols by assuming that channel state information (CSI) is available at the transmitter side and CSI takes values from a prescribed finite set of values. In [11], by assuming that partial CSI is available, optimal transmission power in each retransmission round was determined for an H-ARQ protocol by a linear programming method that selects a power value from a set of discrete power levels. Recently, an optimal power transmission strategy was identified for a basic ARQ protocol where the receiver decodes based only on the received signal in each transmission round and the channel changes independently per transmission [12]. A necessary and sufficient condition for the optimal transmission power sequence was found which indicates that power must be increasing in every retransmission. We note that this result is not applicable to slowly fading channels.

In this work, we study advanced H-ARQ protocols in which the destination node decodes an information packet by combining all received signals from previous (re-)transmission rounds. We assume that the source-destination channel experiences slow fading, i.e. the channel does not change during retransmissions of the same information packet. Our goal is to find the optimal power assignment strategy for retransmissions that minimizes the average total transmission power for any given targeted outage probability. First, we derive a set of equations that describe the optimal transmission power values and enable their exact recursive calculation. Interestingly, it turns out that the optimal transmission power assignment sequence is neither increasing nor decreasing; its form depends on given total power budget and targeted outage performance levels. To reduce calculation complexity, we also develop an approximation to the optimal power sequence that is close to the numerically calculated exact result. The optimal power assignment values reveal that the conventional equal-power assignment is far from optimal. As an example, for a targeted outage probability of 10^{-3} and maximum number of transmissions $L = 2$, the average total transmission power based on the optimal power assignment is 9dB less than that of using the common equal-power scheme.

II. System Model and Problem Formulation

We consider an H-ARQ transmission protocol implemented between a source node and a destination node as illustrated in Fig. 1. The H-ARQ transmission scheme operates as follows. First, the source transmits an information packet to the destination and the destination indicates success or failure of receiving the packet by feeding back a single bit of acknowledge (ACK) or negative-acknowledgement (NACK), respectively. The feedback channel is assumed error-free. Then, if NACK is received by the source and the maximum number of transmissions L is not reached, the source retransmits the packet at a potentially different transmission power to be determined/optimized. If ACK is received by the source or the maximum transmission number L is reached, the source begins transmission of a new information packet. In each retransmission round, the destination attempts to decode an information packet by

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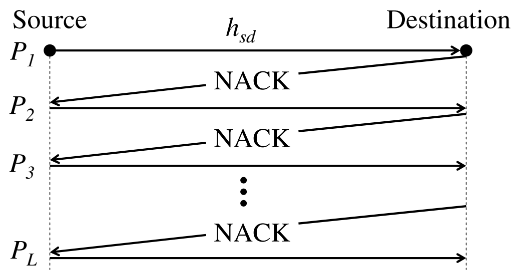

Fig. 1. Illustration of a hybrid-ARQ protocol with transmission power P_l per (re-)transmission, $1 \leq l \leq L$.

combining received signals from all previous transmission rounds by standard maximal-ratio-combining (MRC) [13]. If the destination still cannot decode an information packet after L transmission rounds, then an outage is declared which means that the signal-to-noise ratio (SNR) of the combined received signals at the destination is below a required SNR.

The H-ARQ transmission scheme can be modeled as follows. The received signal $y_{sd,l}$ at the destination at the lth transmission round can be written as

$$y_{sd,l} = \sqrt{P_l}\ h_{sd}\ x_s + \eta_{sd,l}, \quad l = 1, 2, \cdots, L, \tag{1}$$

where x_s is the transmitted information symbol from the source, P_l is the transmission power used by the source at the lth transmission round, h_{sd} is the source-destination channel coefficient, and $\eta_{sd,l}$ is additive noise. The channel coefficient h_{sd} is modeled as zero-mean complex Gaussian random variable with variance σ_{sd}^2. The channel is assumed to be *quasi-static*, i.e. the channel remains fixed during retransmissions of the same packet and may change independently when a new information packet is to be transmitted. The source-destination channel is assumed to be known at the destination side, but unknown at the source side. The additive noise contribution $\eta_{sd,l}$ is modeled as a zero-mean complex Gaussian random variable with variance $\mathcal{N}_0$.

At the destination side, the receiving node combines the received signals from all previous retransmission rounds and jointly decodes the information packet. The overall SNR of the combined signal at the destination at the lth $(1 \leq l \leq L)$ retransmission round is

$$\gamma_{sd,l} = \frac{\sum_{i=1}^{l} P_i |h_{sd}|^2}{\mathcal{N}_0}. \tag{2}$$

Thus, for a targeted SNR γ_0, the probability of the event that the destination cannot decode correctly after l transmission rounds can be calculated, for a Rayleigh channel with variance σ_{sd}^2, to be

$$p^{out,l} = \Pr\left[\gamma_{sd,l} < \gamma_0\right] = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l} P_i}}. \tag{3}$$

Set $p^{out,0} = 1$. Then, the probability that the H-ARQ protocol stops at the lth, $1 \leq l < L$, transmission round is $p^{out,l-1} - p^{out,l}$, which means the destination cannot decode correctly at the $(l-1)$th round, but succeeds at the lth round.

Our goal is to find an optimal power allocation sequence $\mathbf{P} = [P_1, P_2, ..., P_L]$ for the H-ARQ protocol such that the average total transmission power for the protocol to deliver an information packet is minimized. Since the probability that the protocol succeeds exactly at the lth round is $p^{out,l-1} - p^{out,l}$ and the corresponding total transmission power is $P_1 + P_2 + \cdots + P_l$, the average total transmission power of the H-ARQ protocol can be expressed as

$$\bar{P} = \sum_{l=1}^{L-1} \left(p^{out,l-1} - p^{out,l}\right) \sum_{i=1}^{l} P_i + p^{out,L-1} \sum_{i=1}^{L} P_i. \tag{4}$$

Note that the protocol stops retransmissions after the Lth round no matter whether decoding at the Lth round is successful or not. For the H-ARQ protocol with a targeted outage probability p_0, the problem of optimal power assignment can be formulated as follows:

$$\begin{aligned} \min \quad & \bar{P} \quad \text{with respect to } P_1, P_2, \cdots, P_L \geq 0 \\ \text{subject to} \quad & p^{out,L} \leq p_0 \end{aligned} \tag{5}$$

where $\bar{P}$ is specified in (4).

III. Optimal Power Sequence

The average total transmission power in (4) can be rewritten by switching the summation order as follows

$$\bar{P} = \sum_{i=1}^{L} P_i \left[\sum_{l=i}^{L-1} \left(p^{out,l-1} - p^{out,l}\right) + p^{out,L-1}\right] \tag{6}$$

where we consider the summation in terms of the index i, $1 \leq i \leq L$, first. Since $\sum_{l=i}^{L-1} \left(p^{out,l-1} - p^{out,l}\right) = p^{out,i-1} - p^{out,L-1}$, the average total transmission power can be simplified to

$$\bar{P} = P_1 + \sum_{l=2}^{L} P_l\ p^{out,l-1}. \tag{7}$$

The constraint in (5) means that with a targeted SNR γ_0, the outage probability of the H-ARQ protocol with L retransmissions should be no more than the specified outage probability value p_0, i.e.

$$p^{out,L} = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{L} P_i}} \leq p_0. \tag{8}$$

Define $P_0 \triangleq \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}$. Then, the constraint takes the form $\sum_{l=1}^{L} P_l \geq P_0$ and the optimization problem in (5) becomes

$$\begin{aligned} \min_{P_1,\cdots,P_L \geq 0} \quad & \bar{P} = P_1 + \sum_{l=2}^{L} P_l \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}}\right] \\ \text{subject to} \quad & \sum_{l=1}^{L} P_l \geq P_0. \end{aligned} \tag{9}$$

Next, we relax temporarily the non-negative condition on P_l, $l = 1, 2, \cdots, L$, consider the sum-power constraint with equality, and form the Lagrangian objective function

$$\mathcal{L}(\mathbf{P}, \lambda) = P_1 + \sum_{l=2}^{L} P_l \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}}\right] + \lambda \left[\sum_{l=1}^{L} P_l - P_0\right]. \tag{10}$$

Taking the derivative of $\mathcal{L}(\mathbf{P},\lambda)$ with respect to λ and setting it equal to zero, we have the power constraint as $\sum_{l=1}^{L} P_l - P_0 = 0$. The derivatives of $\mathcal{L}(\mathbf{P},\lambda)$ with respect to P_k, $k = 1, 2, \cdots, L$, are

$$\frac{\partial \mathcal{L}}{\partial P_k} = \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-1} P_i}}\right] - \sum_{l=k+1}^{L} \frac{P_l \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{l-1} P_i\right)^2} e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}} + \lambda. \tag{11}$$

According to $\frac{\partial \mathcal{L}}{\partial P_1} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_2} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_1} - \frac{\partial \mathcal{L}}{\partial P_2} = \left[1 - \frac{P_2 \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1^2}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1}} = 0, \tag{12}$$

which means

$$P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}. \tag{13}$$

For any $k = 3, 4, \cdots, L$, according to $\frac{\partial \mathcal{L}}{\partial P_{k-1}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_k} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{k-1}} - \frac{\partial \mathcal{L}}{\partial P_k} = -\, e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-2} P_i}} + \left[1 - \frac{P_k \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)^2}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-1} P_i}} = 0, \tag{14}$$

which implies

$$P_k = \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \left[1 - e^{-\frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)\left(\sum_{i=1}^{k-2} P_i\right)}}\right], \tag{15}$$

for any $k = 3, 4, \cdots, L$. We can easily verify now that the obtained power values are positive and $\bar{P}$ cannot be further minimized with strict inequality in (9). We summarize the above discussion in the following theorem.

Theorem 1: In the H-ARQ transmission protocol, to minimize the average total transmission power, the optimal transmission power P_k at the kth, $1 \le k \le L$, transmission round must satisfy

$$P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}, \tag{16}$$

$$P_k = \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \left[1 - e^{-\frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)\left(\sum_{i=1}^{k-2} P_i\right)}}\right], \tag{17}$$

for $k = 3, 4, \cdots, L$, and

$$P_1 + P_2 + \cdots + P_L = P_0, \tag{18}$$

where $P_0 \triangleq \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}$, γ_0 is the required SNR for correct decoding, and p_0 is the targeted H-ARQ outage probability. □

From Theorem 1, we can see that the optimal transmission power values can be calculated recursively. According to (16) and (17), optimal transmission power value $P_k, k = 2, 3, \cdots, L$, can be calculated based on $P_1, P_2, \cdots, P_{k-1}$. So for any given power P_1, all other transmission power $P_k, k = 2, 3, \cdots, L$, can be determined. The optimal initial power P_1 can be numerically found by (18). When $L = 2$, for example, the optimal transmission power numbers P_1 and P_2 are $P_1 = \frac{2P_0}{1+\sqrt{1+\frac{4\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}}}$, and $P_2 = \frac{\gamma_0 \mathcal{N}_0}{4\sigma_{sd}^2}\left(\sqrt{1+\frac{4\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}} - 1\right)^2$. From the theorem, we also observe that the optimal power assignment sequence is neither increasing nor decreasing. For example, if $P_1 < \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$, then the optimal transmission power P_2 is less than P_1. On the other hand, if $P_1 > \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$, then the optimal transmission power P_2 is larger than P_1. This phenomenon is fundamentally different from the case in [12] where the optimal transmission power values are increasing in every retransmission.

In the remaining of this section, to reduce calculation complexity we further present a simple and tight approximation for the optimal transmission power sequence. Since $1 - e^{-x} \approx x$ for small x, for any $k = 3, 4, \cdots, L$, the optimal transmission power P_k in (17) can be approximated as

$$\begin{aligned} P_k &\approx \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \times \frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)} \\ &= P_{k-1} + \frac{P_{k-1}^2}{\sum_{i=1}^{k-2} P_i}, \quad k = 3, 4, \cdots, L. \end{aligned} \tag{19}$$

Substituting $P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}$ into the above approximation, we have

$$P_3 \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right). \tag{20}$$

By induction, we can show that

$$P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2}, \tag{21}$$

for any $k = 3, 4, \cdots, L$. Substituting (21) into (18), we have a sum-power constraint as follows

$$P_1 + \sum_{k=2}^{L} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2} = P_0 \tag{22}$$

or equivalently

$$P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L} = P_0. \tag{23}$$

We summarize the above discussion in the form of the following theorem.

Theorem 2: In the H-ARQ transmission protocol, the optimal transmission power at each round can be approximated as

$$P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2} \tag{24}$$

for $k = 2, 3, \cdots, L$ where P_1 is determined by the constraint

$$P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L-1} = P_0. \tag{25}$$

From Theorem 2, we can see that the optimal transmission power P_1 can be directly determined by (25). Then, P_1 can be used to determine all other optimal transmission power values $P_k, k = 2, 3, \cdots, L$. Based on (25), the optimal transmission power P_1 is bounded by

$$\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}\left[\left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}\right)^{\frac{1}{L}} - \frac{L-1}{L}\right] < P_1 < \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}\left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}\right)^{\frac{1}{L}}. \tag{26}$$

In Fig. 2 we show comparisons of the approximation of the optimal transmission power values by Theorem 2 with the exact optimized values by Theorem 1. We assumed $\sigma_{sd}^2 = 1$ and $\mathcal{N}_0 = 1$. The targeted SNR is $\gamma_0 = 10$ dB and the required outage performance is $p_0 = 10^{-3}$. The maximum number of transmission rounds is $L = 5$. We can see that the approximations of the optimal transmission power values are very close to the exact optimal values. For comparison, we also include in the figures the transmission power of equal-power assignment protocols. We observe that in the first few retransmission rounds, the optimal power assignment strategy assigns significantly less transmission power compared to the equal-power assignment strategy.

Finally, utilizing the tight approximation of the optimal transmission power sequence, the average total transmission power of the H-ARQ protocol accounting for requested retransmissions can be approximated as follows

$$\bar{P}_{opt} \approx P_1 + \sum_{l=2}^{L} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{l-2} \times \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1}\left(1+\frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{-l+2}}\right], \tag{27}$$

which depends only on P_1. Moreover, if $P_1 > \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$, then using the approximation $1 - e^{-x} \approx x$ for small x, the average total transmission power across requested retransmissions can be further approximated as

$$\bar{P}_{opt} \approx P_1 + \sum_{l=2}^{L} P_1 = L P_1. \tag{28}$$

For comparison purposes, if we consider an equal power assignment strategy, the average total transmission power of the H-ARQ protocol can be obtained as

$$\bar{P}_{equ} = P_0 - \frac{P_0}{L}\sum_{l=2}^{L} e^{-\frac{L}{l-1}\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_0}}, \tag{29}$$

in which a constant power P_0/L is used in each transmission.

IV. Numerical Results

In this section, by numerical calculation we compare the power efficiency of the optimal power assignment strategy derived in this work and the conventional equal-power assignment approach. We assume that the variance of the channel h_{sd} is $\sigma_{sd}^2 = 1$ and the noise variance is $\mathcal{N}_0 = 1$.

The average total transmission power of the two power assignment schemes are shown in Figs. 3 and 4 for the cases of $L = 2$ and $L = 5$, respectively, as a function of the targeted SNR γ_0. The required outage performance of the H-ARQ protocol is set at $p_0 = 10^{-3}$. When $L = 2$, from Fig. 3 we observe that the optimal power assignment saves about 9 dB in average total transmission power compared to the equal-power H-ARQ. When $L = 5$, from Fig. 4 we can see that the optimal power assignment shows an 11 dB gain compared to the equal-power assignment scheme. In both figures, the exact calculation of the average total transmission power is based on Theorem 1 and the approximation is based on Theorem 2. Again, the approximations match tightly with the exact calculation results.

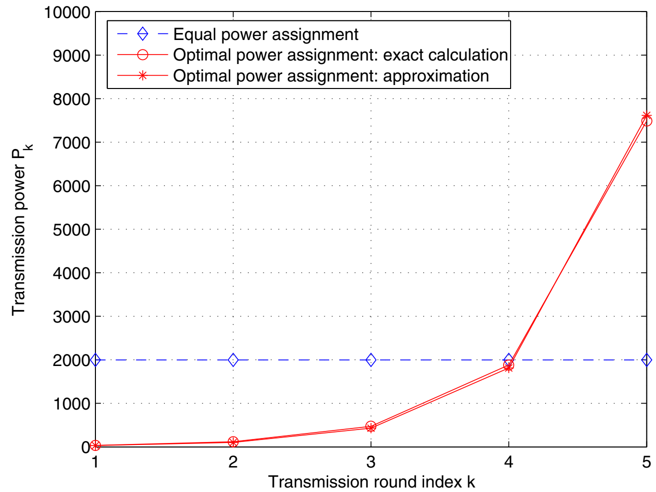

Fig. 2. Transmission power sequence of equal and optimal power assignment strategies with $L = 5$, $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$.

Figs. 5 and 6 present the average total transmission power required for different targeted outage probability values where $\gamma_0 = 10$ dB. The maximum number of transmission rounds is $L = 2$ in Fig. 5 and $L = 5$ in Fig. 6. From the two figures, we can see that for an outage performance of $p_0 = 10^{-4}$, the power savings of the optimal power assignment scheme compared to the equal-power assignment scheme are 15 dB when $L = 2$ and 19 dB when $L = 5$. The lower the required outage probability, the more important optimization of the power sequence becomes. Similarly, with the same targeted outage performance, the larger the number of retransmission rounds allowed in the protocol, the larger the performance gain between the optimal power assignment scheme and the equal-power assignment scheme.

V. Conclusion

In this paper, we determined an optimal power assignment sequence for the H-ARQ transmission protocol in quasi-static Rayleigh fading channels. Both an exact recursive expression and a direct tight approximation of the optimal transmission power were presented. The optimal power assignment values reveal that conventional equal-power assignment is far from optimal. For example, for a targeted outage performance of

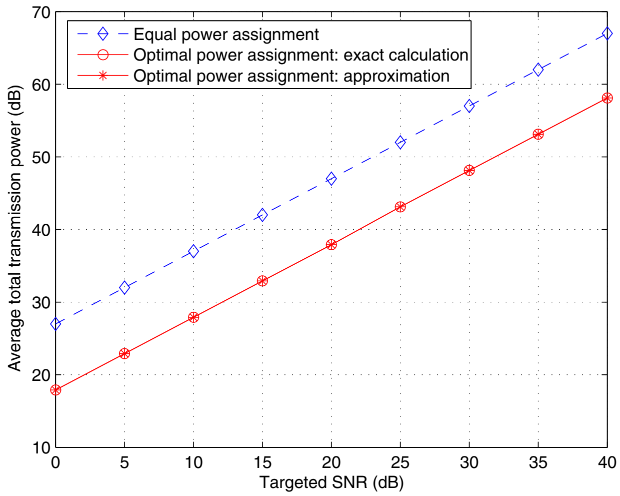

Fig. 3. Average total transmission power of the equal and optimum power assignment. $L = 2$, $p_0 = 10^{-3}$.

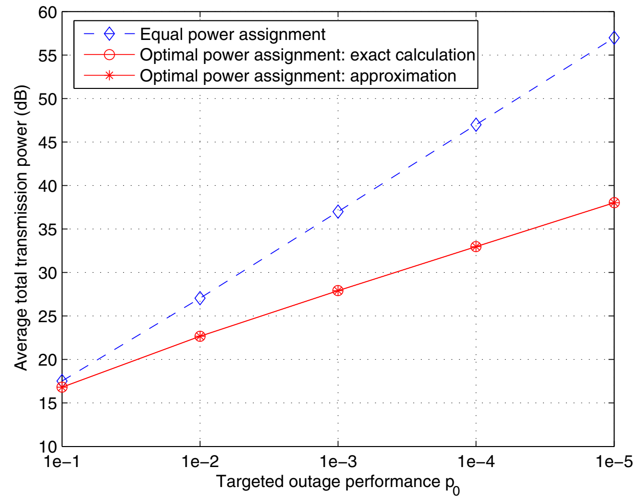

Fig. 5. Average total transmission power of the equal and optimum power assignment. $L = 2$, $\gamma_0 = 10$ dB.

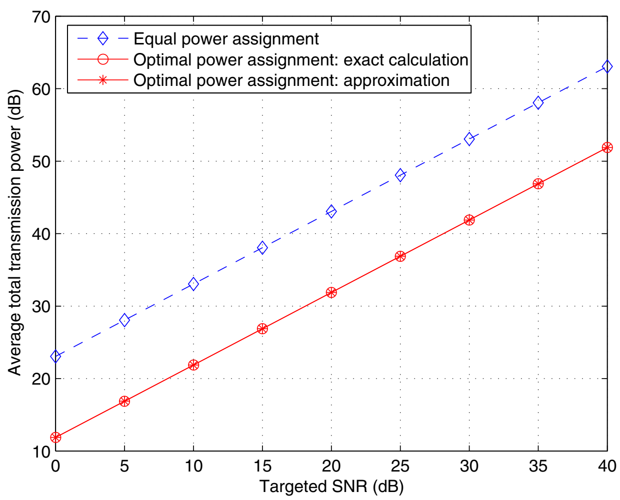

Fig. 4. Average total transmission power of the equal and optimum power assignment. $L = 5$, $p_0 = 10^{-3}$.

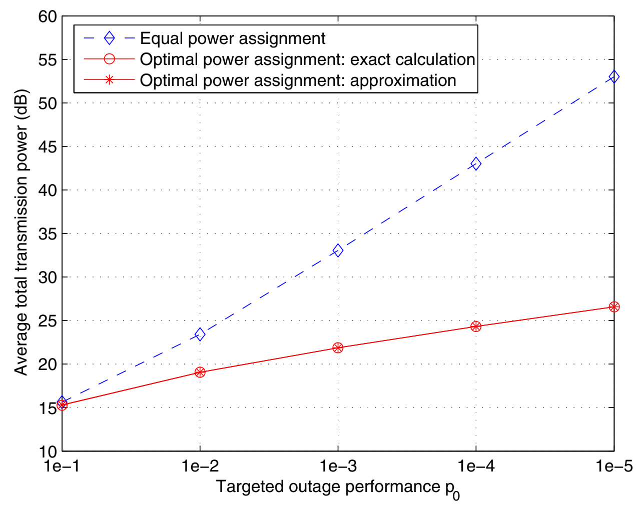

Fig. 6. Average total transmission power of the equal and optimum power assignment. $L = 5$, $\gamma_0 = 10$ dB.

10^{-5} and maximum number of transmissions $L = 5$, the average total transmission power by the optimum assignment is about 27dB less than that of using equal-power assignment.

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