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The Optimal Power Assignment for Cooperative Hybrid-ARQ Relaying Protocol

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***Abstract*— In this work, we consider the problem of assigning optimal transmission power sequence for cooperative hybrid automatic-repeat-request (H-ARQ) relaying protocol over quasi-static Rayleigh fading channels. We try to determine the optimal power sequence by minimizing the average total transmission power that we analyzed in our previous work in [10]. However, the closed-form expression of the average total power consumption of the cooperative H-ARQ relaying protocol is complicated in general, so we develop first in this work a simple approximation of the average total transmission power that is tight at high SNR. Then, based on the asymptotically tight approximation, we are able to identify the sequence of power values that minimizes the average total power consumption of the cooperative H-ARQ relaying protocol for any given targeted outage probability. In particular, we derive a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. When the maximum number of (re)transmissions allowed in the protocol is $L = 2$, we have a closed-form result for the optimal transmission power sequence. The optimal power assignment solution reveals that conventional equal power assignment scheme is not optimal in general. Extensive simulation and numerical results are provided to illustrate and validate the theoretical results.**

Index Terms:[1] **Hybrid automatic-repeat-request (H-ARQ) protocol, cooperative H-ARQ relaying, optimal power assignment, outage probability, wireless networks.**

I. Introduction

Cooperative wireless networks can substantially increase network reliability as each user's information may be jointly delivered to its destination with the assistance of cooperative users in the networks [1]–[3]. On the other hand, automatic-repeat-request (ARQ) protocols have been long-time used to enable reliable data packet transmissions in data link control [4]–[6]. In basic ARQ protocols, a receiver decodes an information packet based only on the received signal in each round [4], [5], while in more advanced ARQ protocols, a receiver may decode an information packet by combining received signals from all previous rounds, resulting in so-called hybrid ARQ (H-ARQ) protocols [6].

It is a natural idea to exploit H-ARQ protocols in conjunction with the cooperative communication concept to jointly enhance link connectivity and network reliability. If the destination requests retransmission, the nearby users may also assist forwarding the signal alongside the source's retransmission. In wireless links formed by wireless devices with limited power resources, power efficiency is a key research matter in the optimization of ARQ retransmission protocols [7]–[9]. In [7], optimal transmission power in each (re)transmission round was determined for an H-ARQ protocol by a linear programming method that selects a power value from a set of discrete power levels. In [8], without assuming channel state information (CSI) available at the transmitter side, an optimal power transmission strategy was identified for a basic ARQ protocol where the receiver decodes based only on the received signal in each round. In [9], by a recursive calculation, an optimal power assignment sequence for an H-ARQ protocol was determined in quasi-static Rayleigh fading, in which the channel does not change during (re)transmissions of the same information packet.

Note that, while the average total power consumption needed in the delivery of each information packet has been well understood for the conventional non-cooperative H-ARQ protocols [7]–[9], the study of cooperative H-ARQ counterpart turns out very challenging with limited results [10]. In this work, we try to determine the optimal power sequence assignment for the cooperative H-ARQ relaying protocol by minimizing the average total power consumption of the protocol that we developed in [10]. However, the closed-form expression of the average total power consumption in [10] is complicated in general, so we develop in this work a simple approximation of the average total transmission power that is tight at high SNR. Then, based on the asymptotically tight approximation, we are able to identify the sequence of power values that minimizes the average total power consumption of the cooperative H-ARQ relaying protocol for any given targeted outage probability. In particular, we derive a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. When the maximum number of (re)transmissions allowed in the cooperative H-ARQ protocol is $L = 2$, we have a closed-form result for the optimal transmission power sequence. The optimal power assignment solution reveals that conventional equal power assignment scheme is not optimal in general. Extensive numerical results are provided to illustrate and validate the theoretical results.

II. System Model

For simplicity in presentation, we consider a cooperative H-ARQ relaying model with one source, one relay, and one destination, as illustrated in Fig. 1. The H-ARQ relay scheme operates as follows. First, the source broadcasts an informa-

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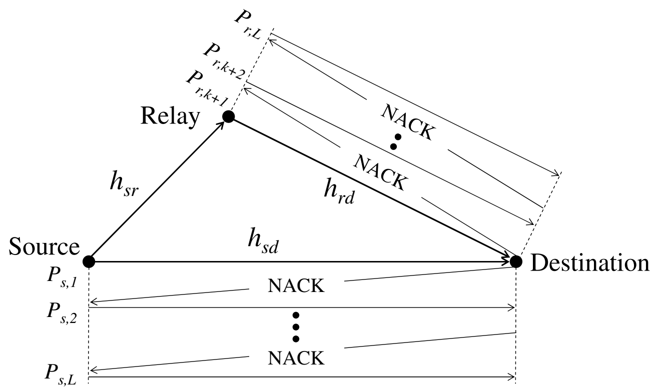

Fig. 1. A cooperative H-ARQ protocol with source (re)transmission power $P_{s,m}$ $(1 \leq m \leq L)$ and relay power $P_{r,n}$ $(k+1 \leq n \leq L)$ when the relay decodes correctly at the k-th round and starts cooperating at round $(k+1)$.

tion packet to the destination and the relay. The destination sends a single bit of acknowledgement (ACK) or negative-acknowledgement (NACK) indicating success or failure of receiving the packet, respectively, to both the source and the relay. The ACK/NACK feedback is assumed to be detected error-free at the source and the relay. If ACK is received by the source or retransmission reaches the maximum number of rounds L, the source begins transmission of a new information packet. If NACK is received by the source and the maximum number of rounds L is not reached, the source retransmits the packet at a potentially different transmission power. If the relay decodes successfully ahead of the destination, the relay starts cooperating with the source by forwarding the packet to the destination by using a space-time transmission, for example the Alamouti scheme [11]. The destination combines the signals from the source and the signals from the relay and jointly decodes the information packet. If the destination still cannot decode an information packet after L transmission rounds, an outage event is declared which means that the signal-to-noise (SNR) of the combined received signals is below a required SNR.

The cooperative H-ARQ relay scheme can be modeled as follows. The received signal $y_{r,m}$ *at the relay* at the m-th $(1 \leq m \leq L)$ H-ARQ (re)transmission round can be modeled as

$$y_{r,m} = \sqrt{P_{s,m}} h_{sr} x_s + \eta_{r,m}, \tag{1}$$

where $P_{s,m}$ is the source transmitted power at the m-th H-ARQ (re)transmission round, h_{sr} is the coefficient of the source-relay channel, x_s is the transmitted information packet from the source, and $\eta_{r,m}$ is additive noise. If the relay is not involved in forwarding, the received signal $y_{d,m}$ *at the destination* at the m-th H-ARQ (re)transmission round is

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \eta_{d,m}, \tag{2}$$

where h_{sd} is the source-destination channel coefficient.

If the relay decodes the packet from the source successfully, it helps in forwarding it to the destination using the Alamouti scheme. It is assumed that the relay knows the codeword of the packet. Specifically, if the packet is partitioned into two parts as $x_s = [x_{s,1}\ x_{s,2}]$, then the relay forwards a corresponding vector $x_r = \left[-x_{s,2}^*\ x_{s,1}^*\right]$. The received signal $y_{d,m}$ *at the destination* at the m-th H-ARQ (re)transmission round with relay forwarding can be written as

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \sqrt{P_r} h_{rd} x_r + \eta_{d,m}, \tag{3}$$

where P_r is the relay transmitted power which is assumed to be fixed over all (re)transmission rounds[2] and h_{rd} is the channel coefficient from the relay to the destination. At the destination, the packet x_s can be recovered based on the orthogonal structure of the Alamouti code. The destination combines the received signals from all (re)transmission rounds and jointly decodes the information packet based on maximal ratio combining (MRC) [12]. We consider quasi-static Rayleigh fading channels, i.e., the channel coefficients are assumed to be fixed during (re)transmissions of a packet and may change independently when a new information packet is transmitted. The channel state information is assumed to be known at the receiver side and unknown at the transmitter side. The channel coefficients h_{sd}, h_{sr} and h_{rd} are modeled as independent, zero-mean complex Gaussian random variables with variances $1/\lambda_{sd}$, $1/\lambda_{sr}$ and $1/\lambda_{rd}$, respectively. The noise variances $\eta_{r,m}$ and $\eta_{d,m}$ are modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$.

III. Average Total Transmission Power

When the destination successfully decodes an information packet, the cooperative H-ARQ protocol stops and that may happen at any round $l\,(1 \leq l \leq L)$. Let $\{T_r = k\}$ denote the event that the relay decodes successfully at the k-th round and starts forwarding at round $(k+1)$, for any $k = 1, 2, \cdots, l-1$. Especially, let $\{T_r = l\}$ denote the event that the relay decodes unsuccessfully in the first $l-1$ rounds (in this case, the relay has no participation in message forwarding when the protocol finishes at the l-th round). Furthermore, let $p^{stop,1}$ denote the probability that the H-ARQ protocol stops at the first transmission round $(l = 1)$, and for any $2 \leq l \leq L-1$, let $q_{l,k}$ denote the conditional probability that the H-ARQ protocol stops at the l-th round given that the event $\{T_r = k\}$ occurred. Especially, let $q_{l,l}\,(2 \leq l \leq L-1)$ denote the probability that the protocol stops successfully at the l-th round before the relay can help. Moreover, when $l = L$, let $q_{L,k}$ denote the conditional probability that the protocol stops at the L-th round no matter whether decoding at the L-th round is successful or not under the condition that the relay started forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, L-1$. Especially, let $q_{L,L}$ denote the probability that the protocol stops at the last round regardless of successful decoding or not (in this case the relay has no chance to help).

With the above notation, the average total power consump-

[2] For simplicity in the treatment herein as well as practical system implementation purposes, it is assumed that relay helpers have fixed, non varying, power level across retransmissions.

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tion of the cooperative H-ARQ protocol is [10]

$$\bar{P}(L) = P_{s,1}p^{stop,1} + \sum_{l=2}^{L}\sum_{k=1}^{l} q_{l,k}\Pr\left[T_r = k\right] R_{l,k}, \qquad (4)$$

where $R_{l,k}$ is the transmission power that the source and the relay spend totally up to the l-th round, which is given by

$$R_{l,k} = \Omega_l + (l-k)P_r, \qquad (5)$$

where $\Omega_l = \sum_{m=1}^{l} P_{s,m}$. A closed-form expression of the average total transmission power is given by [10]

$$\bar{P}(L) = P_{s,1}e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1}\sum_{k=1}^{l} C_{l,k} + \sum_{k=1}^{L} D_k, \qquad (6)$$

where

$$\begin{aligned} C_{l,k} &= q_{l,k}\Pr\left[T_r = k\right] R_{l,k} \\ &= (A_1+A_2-A_3)\left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}}\right)\left[\Omega_l + (l-k)P_r\right], \\ &\qquad 1 \le k \le l-1, \\ C_{l,l} &= q_{l,l}\Pr\left[T_r = l\right] R_{l,l} \\ &= \left(e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}}\right)\left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}}\right)\Omega_l, \\ D_k &= q_{L,k}\Pr\left[T_r = k\right] R_{L,k} \\ &= B_{L-1,k}\left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}}\right)\left[\Omega_L + (L-k)P_r\right], \\ &\qquad 1 \le k \le L-1, \\ D_L &= q_{L,L}\Pr\left[T_r = L\right] R_{L,L} \\ &= \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right)\left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right)\Omega_L, \end{aligned}$$

in which

$$A_1 = \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k)P_r}{\Omega_l} - \lambda_{rd}}\left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}}\right), \qquad (7)$$

$$A_2 = e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}}, \qquad (8)$$

$$A_3 = \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k-1)P_r}{\Omega_{l-1}} - \lambda_{rd}}\left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}}\right), \qquad (9)$$

$$B_{i,j} = 1 - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(i-j)P_r}} - \frac{\lambda_{rd}}{\frac{\lambda_{sd}(i-j)P_r}{\Omega_i} - \lambda_{rd}}\left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(i-j)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i}}\right), \qquad (10)$$

in which $\tilde{\gamma}_0 \triangleq \gamma_0\mathcal{N}_0$ and γ_0 is the target SNR. When $L = 2$, the average total transmission power expression $\bar{P}$ in (6) reduces to

$$\bar{P}(L=2) = P_{s,1} + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}}\right)\left(P_{s,2} + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{P_{s,1}}} P_r\right). \qquad (11)$$

The closed-form expression of the average total transmission power in (6) is certainly complicated. In this paper, we try to develop a simple and tight approximation of the average total transmission power which allows us to determine an optimum power assignment strategy for the cooperative H-ARQ protocol. When $L=2$, if we approximate e^{-x} by $1-x$ for small x (corresponding to high SNR), then the average total transmission power $\bar{P}$ in (11) can be approximated as

$$\bar{P}(L=2) \approx P_{s,1} + (P_{s,2} + P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}, \qquad (12)$$

which is tight at high SNR. When $L \ge 3$, however, it is difficult to derive the approximation of $\bar{P}$ directly from (6).

In the following, we first reformulate the average total transmission power $\bar{P}$ in (6) which enables us to develop a tight approximation at high SNR. For simplicity, let us denote $\tilde{C}_{l,k} = q_{l,k}\Pr\left[T_r = k\right]$ for $k = 1, 2, \cdots, l-1$, and $\tilde{C}_{l,l} = q_{l,l}\Pr\left[T_r = l\right]$. Denote $\tilde{D}_k = q_{L,k}\Pr\left[T_r = k\right]$ for $k = 1, 2, \cdots, L-1$, and $\tilde{D}_L = q_{L,L}\Pr\left[T_r = L\right]$. We observe that $C_{l,k} = \tilde{C}_{l,k}\left[\Omega_l + (l-k)P_r\right]$, $C_{l,l} = \tilde{C}_{l,l}\Omega_l$, $D_k = \tilde{D}_k\left[\Omega_L + (L-k)P_r\right]$, and $D_L = \tilde{D}_L\Omega_L$. Therefore, the average total transmission power in (6) can be rewritten by switching the summation order (in terms of $P_{s,m}, 1 \le m \le L$, and P_r first) as follows

$$\bar{P}(L) \triangleq \sum_{m=1}^{L} P_{s,m}Q_m + P_rQ_r, \qquad (13)$$

where

$$Q_1 = e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1}\sum_{k=1}^{l}\tilde{C}_{l,k} + \sum_{k=1}^{L}\tilde{D}_k, \qquad (14)$$

$$Q_m = \sum_{l=m}^{L-1}\sum_{k=1}^{l}\tilde{C}_{l,k} + \sum_{k=1}^{L}\tilde{D}_k, \quad 2 \le m \le L-1, \qquad (15)$$

$$Q_L = \sum_{k=1}^{L}\tilde{D}_k, \qquad (16)$$

$$Q_r = \sum_{l=2}^{L-1}\sum_{k=1}^{l-1}(l-k)\tilde{C}_{l,k} + \sum_{k=1}^{L-1}(L-k)\tilde{D}_k. \qquad (17)$$

We observe that $Q_1 = Q_2 + e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}}$, and for any $m = 2, 3\cdots, L-1$, $Q_m = Q_{m+1} + \sum_{k=1}^{m}\tilde{C}_{m,k}$. The probability Q_m, $1 \le m \le L$, can be expressed in the following lemma. (The lemma can be proved easily by induction, and we omit the proof due to lack of space.)

Lemma 1: The probability Q_m in (14)–(16) has a closed-form expression as follows:

$$Q_m = \begin{cases} 1, & m = 1; \\ 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}, & m = 2; \\ \sum_{k=1}^{m-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}}\left(B_{m-1,k} - B_{m-1,k+1}\right) \\ \qquad + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{m-1}}}\right), & 3 \le m \le L, \end{cases}$$

where $B_{i,j}$ is specified in (10). Furthermore, the probability Q_m can be tightly approximated by

$$Q_m \approx \begin{cases} \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}, & m = 2; \\ \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{m-1}}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{m-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(m-2)P_r}\right), & 3 \le m \le L. \end{cases} \qquad (18)$$

Next, we calculate the probability Q_r in (17). To calculate Q_r more efficiently, we partition Q_r as follows

$$Q_r \triangleq \sum_{i=2}^{L} Q_{r,i}, \qquad (19)$$

where

$$Q_{r,2} = \sum_{l=2}^{L-1} \tilde{C}_{l,1} + \tilde{D}_1, \tag{20}$$

$$Q_{r,i} = \sum_{l=i}^{L-1}\sum_{k=1}^{l-1} \tilde{C}_{l,k} + \sum_{k=1}^{L-2} \tilde{D}_k - \sum_{k=i}^{L-2}\left(\sum_{l=k+1}^{L-1} \tilde{C}_{l,k} + \tilde{D}_k\right), \quad 3 \le i \le L-2, \tag{21}$$

$$Q_{r,L-1} = \sum_{k=1}^{L-2} \tilde{C}_{L-1,k} + \sum_{k=1}^{L-2} \tilde{D}_k, \tag{22}$$

$$Q_{r,L} = \sum_{k=1}^{L-1} \tilde{D}_k. \tag{23}$$

We observe that $Q_{r,L-1} = Q_{r,L} + \sum_{k=1}^{L-2} \tilde{C}_{L-1,k} - \tilde{D}_{L-1}$, and for any $i = 3, 4 \cdots, L-2$, $Q_{r,i} = Q_{r,i+1} + \sum_{k=1}^{i-1} \tilde{C}_{i,k} - \left(\sum_{l=i+1}^{L-1} \tilde{C}_{l,i} + \tilde{D}_i\right)$. The calculation of the probability Q_r can be summarized in the following lemma. The proof is omitted due to lack of space.

Lemma 2: The probability Q_r in (17) has a closed-form expression as follows:

$$Q_r = \sum_{i=2}^{L} Q_{r,i}, \tag{24}$$

where

$$Q_{r,i} = \begin{cases} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}}\left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}\right), & i = 2; \\ \sum_{k=1}^{i-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}}\left(B_{i-1,k} - B_{i-1,k+1}\right) \\ \quad + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}}\left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}}\right) \\ \quad - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right)\left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right), & 3 \le i < L; \\ \sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}}\left(B_{L-1,k} - B_{L-1,k+1}\right) \\ \quad + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right), & i = L, \end{cases}$$

in which $B_{i,j}$ is given in (10). Furthermore, the probability Q_r can be tightly approximated by

$$Q_r \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{i=3}^{L} \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-2)P_r}\right). \tag{25}$$

Based on Lemmas 1 and 2, the average total transmission power $\bar{P}(L)$ in (13) can be approximated as follows.

Theorem 1: In the cooperative H-ARQ relaying protocol, the average total transmission power can be tightly approximated at high SNR scenario as

$$\bar{P}(L) \approx P_{s,1} + (P_{s,2} + P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}} + \sum_{l=3}^{L}(P_{s,l} + P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r}\right). \tag{26}$$

IV. Optimal Transmission Power Assignment

In this section, we determine an optimal power assignment strategy for the cooperative H-ARQ relaying protocol based on the asymptotically tight approximation of the average total power consumption developed in the previous section.

Before we formulate the problem of finding the optimal transmission power sequence, we need to derive the outage probability of the cooperative H-ARQ relaying protocol with maximum L (re)transmission rounds. Let $p_{T_r=k}^{out,L}$ denote the conditional probability that the destination decodes an information packet unsuccessfully after L (re)transmission rounds given that the event $\{T_r = k\}$ occurred. In other words, $p_{T_r=k}^{out,L}$ is the outage probability at the destination in spite of the fact that the relay started forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, L-1$. Therefore, the outage probability of the cooperative H-ARQ relay scheme after L rounds can be represented as

$$p^{out,L} = \sum_{k=1}^{L} p_{T_r=k}^{out,L} \cdot \Pr\left[T_r = k\right]. \tag{27}$$

Similar to the result in Lemma 1, the outage probability $p^{out,L}$ can be specified as

$$p^{out,L} = \sum_{k=1}^{L-1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}}\left(B_{L,k} - B_{L,k+1}\right) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}}\right), \tag{28}$$

where $B_{i,j}$ is given in (10). Similar to the approximation in (18), we may further approximate $p^{out,L}$ at high SNR as

$$p^{out,L} \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r}\right). \tag{29}$$

In the following, we determine an optimal power sequence $\mathbf{P} = [P_{s,1}, P_{s,2}, \cdots, P_{s,L}; P_r]$ for the cooperative H-ARQ relay protocol such that the average total transmission power of the protocol to deliver an information packet is minimized. We assume that the relay transmission power P_r is fixed over all retransmission rounds. For the H-ARQ relay protocol with a targeted outage probability p_0, the problem of finding optimal power assignment per round can be formulated as

$$\begin{aligned} &\min \quad \bar{P} \quad \text{with respect to } P_{s,1}, P_{s,2}, \cdots, P_{s,L}; P_r \ge 0 \\ &\text{subject to} \quad p^{out,L} \le p_0 \end{aligned} \tag{30}$$

where $\bar{P}$ and $p^{out,L}$ are specified in (6) and (28), respectively. Since solving the optimization problem with the closed-form expressions in (6) and (28) is not analytically tractable, we use the asymptotically tight approximation results of the average total transmission power and the outage probability in (26) and (29), respectively. Then, the optimization problem is given by

$$\begin{aligned} \min_{P_{s,1},\cdots,P_{s,L};P_r\ge 0} \quad & P_{s,1} + \frac{\lambda_{sd}\tilde{\gamma}_0(P_{s,2}+P_r)}{\Omega_1} \\ & + \sum_{l=3}^{L}\frac{\lambda_{sd}\tilde{\gamma}_0(P_{s,l}+P_r)}{\Omega_{l-1}}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r}\right) \\ \text{subject to} \quad & \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r}\right) \le p_0 \end{aligned} \tag{31}$$

where $\Omega_l = \sum_{m=1}^{l} P_{s,m}$.

It is not difficult to show that the minimization of $\bar{P}$ can be achieved at the boundary of the constraint in (31) (with equality). In the following, we consider a Lagrange multiplier method to find the optimal power sequence. Denote $F_i \triangleq \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-1)P_r}\right)$, then a Lagrangian objective function can be formulated as

$$\begin{aligned}\mathcal{L}\left(\mathbf{P},\lambda\right) = P_{s,1} + (P_{s,2}+P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} \\ + \sum_{l=3}^{L}(P_{s,l}+P_r)F_{l-1} + \lambda\Big[F_L - p_0\Big]. \quad (32)\end{aligned}$$

Let us further denote $G_i \triangleq \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{\Omega_{i-1}^2}$ and $M_i \triangleq \frac{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0^2}{2(i-1)P_r^2}$, then the derivatives of $\mathcal{L}\left(\mathbf{P},\lambda\right)$ with respect to $P_{s,m}$ and P_r are

$$\begin{cases} \frac{\partial\mathcal{L}}{\partial P_{s,1}} = 1 - (P_{s,2}+P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1^2} \\ \qquad - \sum_{l=3}^{L}(P_{s,l}+P_r)\frac{F_{l-1}+G_{l-1}}{\Omega_{l-1}} - \lambda\frac{F_L+G_L}{\Omega_L}, \\ \frac{\partial\mathcal{L}}{\partial P_{s,2}} = \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (P_{s,3}+P_r)\frac{F_2}{\Omega_2} \\ \qquad - \sum_{l=4}^{L}(P_{s,l}+P_r)\frac{F_{l-1}+G_{l-1}}{\Omega_{l-1}} - \lambda\frac{F_L+G_L}{\Omega_L}, \\ \frac{\partial\mathcal{L}}{\partial P_{s,m}} = F_{m-1} - (P_{s,m+1}+P_r)\frac{F_m}{\Omega_m} \\ \qquad - \sum_{l=m+2}^{L}(P_{s,l}+P_r)\frac{F_{l-1}+G_{l-1}}{\Omega_{l-1}} - \lambda\frac{F_L+G_L}{\Omega_L}, \\ \qquad\qquad m = 3,4,\cdots,L-2, \\ \frac{\partial\mathcal{L}}{\partial P_{s,L-1}} = F_{L-2} - (P_{s,L}+P_r)\frac{F_{L-1}}{\Omega_{L-1}} - \lambda\frac{F_L+G_L}{\Omega_L}, \\ \frac{\partial\mathcal{L}}{\partial P_{s,L}} = F_{L-1} - \lambda\frac{F_L}{\Omega_L}, \\ \frac{\partial\mathcal{L}}{\partial P_r} = \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^{L}\left[F_{l-1} - (P_{s,l}+P_r)\frac{M_{l-1}}{\Omega_{l-1}}\right] - \lambda\frac{M_L}{\Omega_L}. \end{cases}$$

Based on $\frac{\partial\mathcal{L}}{\partial P_{s,1}} - \frac{\partial\mathcal{L}}{\partial P_{s,2}} = 0$, we have

$$P_{s,3} = \frac{\Omega_1\Omega_2}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\left[\Omega_1 - (\Omega_2+P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}\right] - P_r.$$

Based on $\frac{\partial\mathcal{L}}{\partial P_{s,2}} - \frac{\partial\mathcal{L}}{\partial P_{s,3}} = 0$, we have

$$P_{s,4} = \frac{\Omega_2\Omega_3}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\left[\Omega_2\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (\Omega_3+P_r)F_2\right] - P_r.$$

For any $m = 5,6,\cdots,L$, according to $\frac{\partial\mathcal{L}}{\partial P_{s,m-2}} - \frac{\partial\mathcal{L}}{\partial P_{s,m-1}} = 0$, we have

$$P_{s,m} = \frac{\Omega_{m-2}\Omega_{m-1}}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\Big[\Omega_{m-2}F_{m-3} - (\Omega_{m-1}+P_r)F_{m-2}\Big] - P_r.$$

Moreover, by taking derivative of $\mathcal{L}\left(\mathbf{P},\lambda\right)$ with respect to λ and setting it equal to zero, we have a constraint $F_L - p_0 = 0$. Furthermore, from $\frac{\partial\mathcal{L}}{\partial P_{s,L}} = 0$ and $\frac{\partial\mathcal{L}}{\partial P_r} = 0$, we have constraints as $F_{L-1} - \lambda\frac{F_L}{\Omega_L} = 0$ and $\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^{L}\left[F_{l-1} - (P_{s,l}+P_r)\frac{M_{l-1}}{\Omega_{l-1}}\right] - \lambda\frac{M_L}{\Omega_L} = 0$. Based on these three constraints, we come up with $\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^{L}\left[F_{l-1} - (P_{s,l}+P_r)\frac{M_{l-1}}{\Omega_{l-1}}\right] = \frac{F_{L-1}M_L}{p_0}$.

When $L = 2$, we can solve the equations directly and obtain closed-form results as: $P_{s,1} = \tilde{\gamma}_0\left(\frac{2\lambda_{sd}^2\lambda_{sr}}{p_0}\right)^{\frac{1}{3}}$, $P_{s,2} = \tilde{\gamma}_0\left(\frac{\lambda_{sd}\lambda_{sr}^2}{2p_0^2}\right)^{\frac{1}{3}}$ and $P_r = \tilde{\gamma}_0\left(\frac{\lambda_{sd}\lambda_{rd}}{2p_0}\right)^{\frac{1}{2}}$. For $L > 2$, we do not have a closed-form solution for the optimal power sequence, but obtain a recursive solution which is summarized in the following theorem.

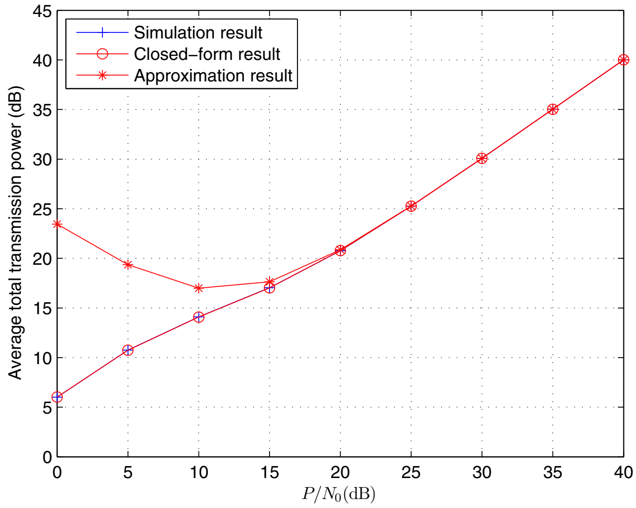

Fig. 2. Average total power consumption per information packet, $P_{s,l} = P_r = P$, $1 \leq l \leq 4$, $\gamma_0 = 10$ dB, $L = 4$.

Theorem 2: In the cooperative H-ARQ relaying protocol, to minimize the average total transmission power, the optimal transmission power sequence $P_{s,1}, P_{s,2}, \cdots, P_{s,L}$ and P_r satisfies the following

$$P_{s,3} = \frac{\Omega_1\Omega_2}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\left[\Omega_1 - (\Omega_2+P_r)\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}\right] - P_r, \quad (33)$$

$$P_{s,4} = \frac{\Omega_2\Omega_3}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\left[\Omega_2\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (\Omega_3+P_r)F_2\right] - P_r, \quad (34)$$

$$P_{s,l} = \frac{\Omega_{l-2}\Omega_{l-1}}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}\Big[\Omega_{l-2}F_{l-3} - (\Omega_{l-1}+P_r)F_{l-2}\Big] - P_r, \quad (35)$$

for $l = 5,6,\cdots,L$, and

$$\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^{L}\left[F_{l-1} - (P_{s,l}+P_r)\frac{M_{l-1}}{\Omega_{l-1}}\right] = \frac{F_{L-1}M_L}{p_0}, \quad (36)$$

where $\Omega_l = \sum_{m=1}^{l} P_{s,m}$, $M_i = \frac{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0^2}{2(i-1)P_r^2}$ and $F_i = \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i}\left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-1)P_r}\right)$. $\square$

From Theorem 2, we observe that for any given power P_r, $P_{s,1}$ and $P_{s,2}$, we can determine power $P_{s,3}$. In general, for any $l = 3,4,\cdots,L$, with power P_r and $P_{s,1}, P_{s,2}, \cdots, P_{s,l-1}$, we can determine power $P_{s,l}$. Thus, for any given initial power P_r, $P_{s,1}$ and $P_{s,2}$, we can determine power $P_{s,l}$ recursively for any $l = 3,4,\cdots,L$. The complexity of finding the optimal power sequence is reduced to that of searching the initial power $P_{s,1}$, $P_{s,2}$ and P_r, i.e. searching over a three-variable space. We can see that in such a way, the calculation complexity of finding the optimal power sequence for the cooperative H-ARQ relaying protocol is fixed regardless of the maximum number of (re)transmission rounds L.

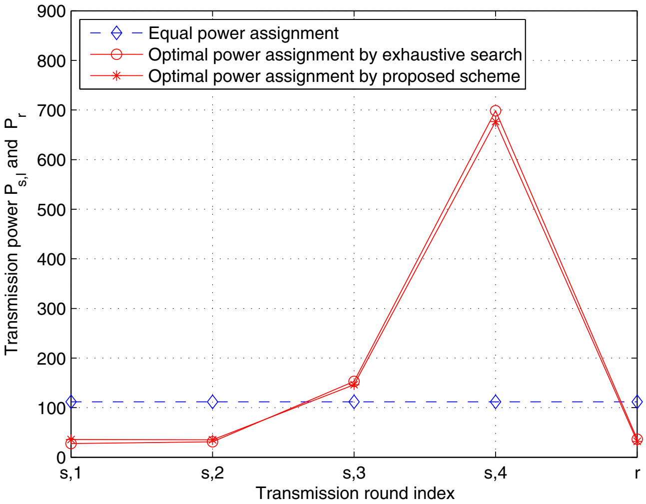

Fig. 3. Transmission power sequence of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 4$.

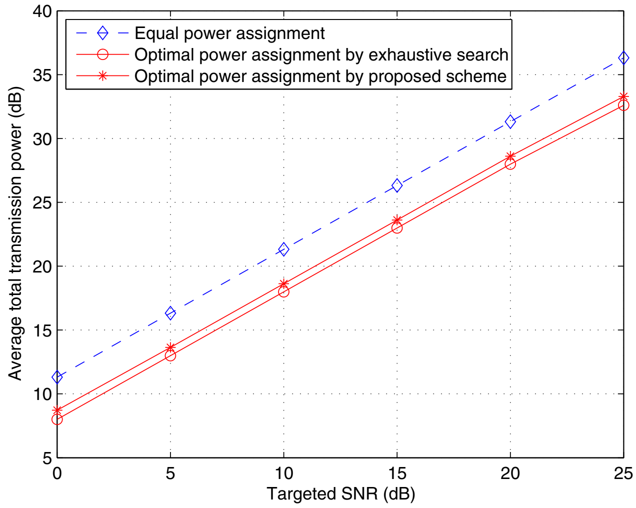

Fig. 4. Average total transmission power of the optimal and equal power assignment strategies with $p_0 = 10^{-3}$, $L = 4$.

V. Numerical and Simulation Results

In this section, we provide some numerical and simulation results to illustrate our theoretical development on the optimal power assignment strategy. In all studies, we assume that the variances of the channels h_{sd}, h_{sr} and h_{rd} are $\frac{1}{\lambda_{sd}} = \frac{1}{\lambda_{sr}} = \frac{1}{\lambda_{rd}} = 1$ and the noise variance is $\mathcal{N}_0 = 1$. We assume that the cooperative H-ARQ relaying protocol allows $L = 4$ maximum (re)transmissions.

In Fig. 2, we plot the average total transmission power of the cooperative H-ARQ relaying protocol by comparing the approximation result in (26), the exact closed-form result in (6) and the simulation result. In this example, we assume $P_{s,l} = P_r = P$, $1 \leq l \leq 4$. We can see that the approximation of the average total transmission power matches closely with the exact closed-form result and the simulation curve at high SNR.

In Fig. 3, we compare the optimal transmission power sequence from Theorem 2 and the exhaustive search result based on the original optimization problem in (30) (without approximation). We can see that the optimal transmission power values resulted from Theorem 2 (solid line with '$*$') match well with the exhaustive search result from the original optimization problem (solid line with '$\circ$'). For comparison, we also include in the figure the transmission power level of the equal power assignment scheme. In Fig. 4, we plot the average total transmission power for both the optimal power assignment scheme and the equal power assignment scheme, with different targeted SNR γ_0 (from 0 dB to 25 dB). We assume the targeted outage probability is $p_0 = 10^{-3}$. We observe that the optimal power assignment scheme saves about 2.6 dB in the average total transmission power compared to the equal power assignment scheme.

VI. Conclusion

In this paper, we developed first a simple approximation of the average total power consumption for the cooperative H-ARQ relaying protocol, which is asymptotically tight at high SNR. Then, based on the asymptotically tight approximation, we determined the optimal transmission power sequence that minimizes the average total power consumption of the protocol for any given targeted outage probability. We derived a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. The optimal power assignment solution reveals that conventional equal power assignment scheme is far from optimal. For example, for a targeted outage performance of $p_0 = 10^{-3}$ and maximum number of (re)transmissions $L = 4$, the optimal power assignment scheme saves about 2.6 dB in the average total power consumption compared to the equal power assignment scheme.

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