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The Average Total Power Consumption of Cooperative Hybrid-ARQ on Quasi-Static Rayleigh Fading Links

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***Abstract*— In this paper, the average total power consumption per information packet is investigated for a cooperative hybrid automatic-repeat-request (H-ARQ) protocol in a quasi-static Rayleigh fading environment. Specifically, a closed-form expression of the average total transmission power is obtained for the cooperative H-ARQ relay protocol, in which the source may use different transmission power level in different (re-)transmission rounds. The closed-form expression is valid for any maximum number of (re-)transmission rounds L allowed by the protocol and may play a key role thereafter in optimizing power allocation. Since the closed-form expression is complicated for large L, an approximation of the average total transmission power is developed which is asymptotically tight at high SNR. Extensive simulation and numerical results are also provided herein to illustrate and validate the theoretical results.**

Index Terms:[1] **Hybrid automatic-repeat-request (H-ARQ) protocol, cooperative H-ARQ, average total transmission power.**

I. Introduction

Cooperative wireless networks can substantially increase network reliability as each user's information may be jointly delivered to its destination with the assistance of cooperative fellow users/nodes in the networks [1]–[3]. On the other hand, automatic-repeat-request (ARQ) protocols have been commonly used to enable reliable data packet transmissions in data link control [4]–[7], in which a receiver requests retransmission when a packet is not correctly received. In a basic ARQ protocol, a receiver decodes an information packet based only on the received signal in each (re-)transmission round [4], [5]. In advanced ARQ protocols, a receiver may decode an information packet by combining received signals from all previous (re-)transmission rounds; such protocols are referred to as hybrid ARQ (H-ARQ) [6], [7].

It is a natural idea, then, to exploit H-ARQ protocols in conjunction with the cooperative communication concept to jointly enhance link connectivity and network reliability. When a source sends a wireless signal to an intended destination, nearby users may also receive the signal. Thus, if the destination requests retransmission, the nearby users may also assist forwarding the signal alongside the source's retransmission. The destination can combine all received signals from source and relays and jointly decode the signals to improve detection performance. Some recent works have studied H-ARQ protocols in the context of cooperative relay networks [8]–[10]. In [8], a general information-theoretic framework of cooperative ARQ relay networks was presented. In [9], [10], upper bounds on the diversity order of a decode-and-forward cooperative ARQ relay scheme were characterized as a means to study the diversity-multiplexing-delay tradeoff. Note that, while the average total power consumption needed in the delivery of each information packet has been well understood for the conventional non-cooperative H-ARQ protocols (see for example [11], [12]), the study of cooperative H-ARQ counterpart has been proven very challenging with no available results until this present work.

In this paper, we investigate the average total power consumption per information packet for a cooperative H-ARQ protocol in which each relay forwards Alamouti-based retransmission signals. We consider a quasi-static Rayleigh fading environment by assuming that the channels do not change during (re-)transmissions of the same information packet and they may change independently when the protocol transmits a new information packet. We develop a new analytical approach and obtain a closed-form expression of the average total transmission power. The closed-form result is valid for any maximum number of retransmission rounds L allowed in the protocol in which the source may use varying transmission power level per round[2]. The closed-form expression may serve, thereafter, as the basis for system designer to optimize power allocation for the cooperative H-ARQ protocol. Since the closed-form expression is rather complicated for large L, we were able to develop simpler approximation of the average total transmission power which is asymptotically tight at high SNR. Extensive simulation and numerical results illustrate and validate our theoretical development.

II. System Model

For simplicity in presentation, we consider a cooperative H-ARQ relaying model with one source, one relay, and one destination, as illustrated in Fig. 1. The H-ARQ relay scheme operates as follows. First, the source broadcasts an information packet to the destination and the relay. The destination sends a single bit of acknowledgement (ACK) or negative-acknowledgement (NACK) indicating success or failure of receiving the packet, respectively, to both the source and the relay. The ACK/NACK feedback is assumed to be detected error-free at the source and the relay. If ACK is received by the source or retransmission reaches the maximum number of rounds L, the source begins transmission of a new information packet. If NACK is received by the source and the maximum

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[2] For simplicity in the treatment herein as well as practical system implementation purposes, it is assumed that relay helpers have fixed, non varying, power level across retransmissions.

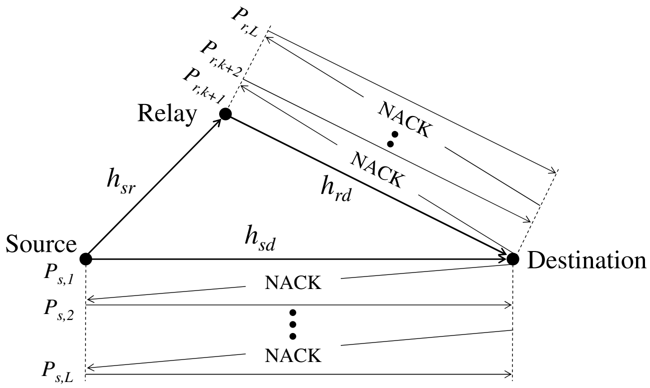

Fig. 1. Illustration of a cooperative H-ARQ protocol with source (re-)transmission power $P_{s,m}$ $(1 \leq m \leq L)$ and relay power $P_{r,n}$ $(k+1 \leq n \leq L)$ when the relay decodes correctly at the k-th round and starts cooperating at round $(k+1)$.

number of rounds L is not reached, the source retransmits the packet at a potentially different transmission power. If the relay decodes successfully ahead of the destination, the relay starts cooperating with the source by forwarding the packet to the destination by using a space-time transmission [9], for example the Alamouti scheme [14]. The destination combines the signals from the source and the signals from the relay and jointly decodes the information packet. If the destination still cannot decode an information packet after L transmission rounds, an outage event is declared which means that the signal-to-noise (SNR) of the combined received signals is below a required SNR.

The cooperative H-ARQ relay scheme can be modeled as follows. The received signal $y_{r,m}$ *at the relay* at the m-th $(1 \leq m \leq L)$ H-ARQ (re-)transmission round can be modeled as

$$y_{r,m} = \sqrt{P_{s,m}} h_{sr} x_s + \eta_{r,m}, \tag{1}$$

where $P_{s,m}$ is the source transmitted power at the m-th H-ARQ (re-)transmission round, h_{sr} is the coefficient of the source-relay channel, x_s is the transmitted information packet from the source, and $\eta_{r,m}$ is additive noise. If the relay is not involved in forwarding, the received signal $y_{d,m}$ *at the destination* at the m-th H-ARQ (re-)transmission round is

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \eta_{d,m}, \tag{2}$$

where h_{sd} is the source-destination channel coefficient.

If the relay decodes the packet from the source successfully, it helps in forwarding it to the destination using the Alamouti scheme. It is assumed that the relay knows the codeword of the packet. Specifically, if the packet is partitioned into two parts as $x_s = [x_{s,1}\ x_{s,2}]$, then the relay forwards a corresponding vector $x_r = \left[-x^*_{s,2}\ x^*_{s,1}\right]$. The received signal $y_{d,m}$ *at the destination* at the m-th H-ARQ (re-)transmission round with relay forwarding can be written as

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \sqrt{P_r} h_{rd} x_r + \eta_{d,m}, \tag{3}$$

where P_r is the relay transmitted power which is assumed to be fixed over all (re-)transmission rounds for simplicity and h_{rd} is the channel coefficient from the relay to the destination. At the destination, the packet x_s can be recovered based on the orthogonal structure of the Alamouti code. The destination combines the received signals from all (re-)transmission rounds and jointly decodes the information packet. We consider quasi-static Rayleigh fading channels, i.e., the channel coefficients are assumed to be fixed during (re-)transmissions of a packet and may change independently when a new information packet is transmitted. The channel state information is assumed to be known at the receiver side and unknown at the transmitter side. The channel coefficients h_{sd}, h_{sr} and h_{rd} are modeled as independent, zero-mean complex Gaussian random variables with variances $1/\lambda_{sd}$, $1/\lambda_{sr}$ and $1/\lambda_{rd}$, respectively. The noise variances $\eta_{r,m}$ and $\eta_{d,m}$ are modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$.

III. Average Total Transmission Power

In this section, we first study the overall SNR at the relay and at the destination resulting from the cooperative H-ARQ retransmissions. Then, we derive the probability of the event that the H-ARQ protocol stops successfully at the l-th $(1 \leq l \leq L)$ round. Finally, we are able to derive the average total transmission power of the cooperative H-ARQ relay protocol.

A. Overall SNR at Relay and Destination

The relay is to combine the received signals from all previous (re-)transmission rounds from the source based on maximal ratio combining (MRC) [13] and jointly decode the information packet. Then, the SNR of the MRC output at the relay at the l-th $(1 \leq l \leq L)$ round is

$$\gamma_{r,l} = \frac{\sum_{m=1}^{l} P_{s,m} |h_{sr}|^2}{\mathcal{N}_0}. \tag{4}$$

The destination combines the received signals from all previous rounds from the source and the relay by MRC and then jointly decodes the information packet. To determine the overall SNR of the combined signal at the destination, we need the following lemma.

Lemma 1: In the cooperative H-ARQ relay scheme, if the relay decodes successfully a packet from the source at the k-th $(1 \leq k < L)$ (re-)transmission round and starts forwarding at round $k+1$, then the overall SNR at the destination at the l-th $(k \leq l \leq L)$ round is given by

$$\gamma_{d,l,k} = \frac{\sum_{m=1}^{l} P_{s,m} |h_{sd}|^2 + (l-k) P_r |h_{rd}|^2}{\mathcal{N}_0}. \tag{5}$$

Proof : Due to space limitations, we give only an outline of the proof. First, let us focus on a single transmission round. Since the packet from the source can be written as $x_s = [x_{s,1}\ x_{s,2}]$ and the relay forwards $x_r = \left[-x^*_{s,2}\ x^*_{s,1}\right]$, so the corresponding received signals $\left[y_{d,m,1}\ y_{d,m,2}\right]^T$ at the destination at the m-th $(1 \leq m \leq L)$ round are

$$\begin{bmatrix} y_{d,m,1} \\ y_{d,m,2} \end{bmatrix} = \begin{bmatrix} \sqrt{P_{s,m}} x_{s,1} & -\sqrt{P_{r,m}} x^*_{s,2} \\ \sqrt{P_{s,m}} x_{s,2} & \sqrt{P_{r,m}} x^*_{s,1} \end{bmatrix} \begin{bmatrix} h_{sd} \\ h_{rd} \end{bmatrix} + \begin{bmatrix} \eta_{d,m,1} \\ \eta_{d,m,2} \end{bmatrix},$$

where $P_{r,m} = \begin{cases} 0, & \text{if } m \le k; \\ P_r, & \text{if } m > k. \end{cases}$ The received signal vector can be rewritten as

$$\underbrace{\begin{bmatrix} y_{d,m,1} \\ y^*_{d,m,2} \end{bmatrix}}_{\triangleq Y_m} = \underbrace{\begin{bmatrix} \sqrt{P_{s,m}} h_{sd} & -\sqrt{P_{r,m}} h_{rd} \\ \sqrt{P_{r,m}} h^*_{rd} & \sqrt{P_{s,m}} h^*_{sd} \end{bmatrix}}_{\triangleq H_m} \underbrace{\begin{bmatrix} x_{s,1} \\ x^*_{s,2} \end{bmatrix}}_{\triangleq \mathrm{x}} + \underbrace{\begin{bmatrix} \eta_{d,m,1} \\ \eta^*_{d,m,2} \end{bmatrix}}_{\triangleq N_m}.$$

Thus, the SNR at the destination at the m-th round is

$$\mathrm{SNR}_m = \frac{\|H_m \mathrm{x}\|_F^2}{\|N_m\|_F^2}. \quad (6)$$

Note that

$$\|H_m \mathrm{x}\|_F^2 = \mathrm{tr}\left(\mathrm{x}\mathrm{x}^{\mathcal{H}} H_m^{\mathcal{H}} H_m\right) = \left(|x_{s,1}|^2 + |x_{s,2}|^2\right)\left(P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2\right).$$

Let us assume that the length and (normalized) power of the packet x_s is $\mathcal{L}$. Then, we have $\|H_m \mathrm{x}\|_F^2 = \mathcal{L}\left(P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2\right)$ and $\|N_m\|_F^2 = \mathcal{L}\mathcal{N}_0$. Thus, the SNR at the destination at the m-th round is

$$\mathrm{SNR}_m = \frac{P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2}{\mathcal{N}_0}, \quad (7)$$

where $P_{r,m} = 0$ if $m \le k$ and $P_{r,m} = P_r$ if $m > k$. Therefore, the overall SNR at the destination by combining all received signals from the first l rounds is

$$\gamma_{d,l,k} = \sum_{m=1}^{l} \mathrm{SNR}_m = \sum_{m=1}^{k} \frac{P_{s,m}|h_{sd}|^2}{\mathcal{N}_0} + \sum_{m=k+1}^{l} \frac{P_{s,m}|h_{sd}|^2 + P_r|h_{rd}|^2}{\mathcal{N}_0}, \quad (8)$$

which leads to the result in the Lemma. □

B. Probability of H-ARQ Stopping at l-th $(1 \le l \le L)$ Round

When the destination successfully decodes an information packet, the cooperative H-ARQ protocol stops and that may happen at any round. In this subsection, we derive the probability of the event that the protocol stops at round l, $1 \le l \le L$.

Let $\{T_r = k\}$ denote the event that the relay decodes successfully at the k-th round and starts forwarding at round $(k+1)$, for any $k = 1, 2, \cdots, l-1$. Especially, let $\{T_r = l\}$ denote the event that the relay decodes unsuccessfully in the first $l-1$ rounds (in this case, the relay has no participation in message forwarding when the protocol finishes at the l-th round). Furthermore, let $p^{stop,1}$ denote the probability that the H-ARQ protocol stops at the first transmission round $(l = 1)$, which means that the destination decodes successfully at round one. When $2 \le l \le L-1$, let $q_{l,k}$ denote the conditional probability that the H-ARQ protocol stops at the l-th round given that the event $\{T_r = k\}$ occurred. In other words, $q_{l,k}$ is the probability of H-ARQ stopping at the l-th round under the condition that the relay started forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, l-1$. Especially, let $q_{l,l}$ $(2 \le l \le L-1)$ denote the probability that the protocol stops successfully at the l-th round before the relay can help. Moreover, when $l = L$, let $q_{L,k}$ denote the conditional probability that the protocol stops at the L-th round no matter whether decoding at the L-th round is successful or not under the condition that the relay started forwarding at the $(k+1)$-th round for any $k = 1, 2, \cdots, L-1$. Especially, let $q_{L,L}$ denote the probability that the protocol stops at the last round regardless of successful decoding or not (in this case again the relay has no chance to help).

With the above notation, the average total transmission power of the cooperative H-ARQ protocol can be given by

$$\bar{P}(L) = P_{s,1}p^{stop,1} + \sum_{l=2}^{L}\sum_{k=1}^{l} q_{l,k} \Pr\left[T_r = k\right] R_{l,k}, \quad (9)$$

where $R_{l,k}$ is the transmission power that the source and the relay spend totally up to the l-th round, which is given by

$$R_{l,k} = \sum_{m=1}^{l} P_{s,m} + (l-k)P_r. \quad (10)$$

First, we calculate the probability $\Pr\left[T_r = k\right]$ in (9). For any given targeted SNR γ_0, the probability that the relay decodes the packet successfully at the first round $(T_r = 1)$ is

$$\Pr\left[T_r = 1\right] = \Pr\left[\frac{P_{s,1}|h_{sr}|^2}{\mathcal{N}_0} \ge \gamma_0\right] = e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{P_{s,1}}}, \quad (11)$$

where $\tilde{\gamma}_0 \triangleq \gamma_0\mathcal{N}_0$. For any $T_r = k$, $k = 2, 3, ..., l-1$, the probability that the relay decodes successfully at the k-th round, which means that the overall SNR is below the targeted SNR γ_0 until the $(k-1)$-th round but above γ_0 at the subsequent k-th round, is calculated as

$$\Pr\left[T_r = k\right] = \Pr\left[\gamma_{r,k-1} < \gamma_0, \gamma_{r,k} \ge \gamma_0\right] = e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k} P_{s,m}}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k-1} P_{s,m}}}. \quad (12)$$

When $T_r = l$, we have

$$\Pr\left[T_r = l\right] = \Pr\left[\gamma_{r,l-1} < \gamma_0\right] = 1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}}}. \quad (13)$$

Next, we calculate the conditional probability $q_{l,k}$ in (9). When $l = 1$, the probability that the protocol stops at the first round is

$$p^{stop,1} = \Pr\left[\frac{P_{s,1}|h_{sd}|^2}{\mathcal{N}_0} \ge \gamma_0\right] = e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}}. \quad (14)$$

When $2 \le l \le L-1$, we consider two scenarios: (i) $k = 1, 2, ..., l-1$, and (ii) $k = l$. For any $k = 1, 2, ..., l-1$, the conditional probability $q_{l,k}$ is given by

$$q_{l,k} = \Pr\left[\gamma_{d,l-1,k} < \gamma_0, \gamma_{d,l,k} \ge \gamma_0\right] = \Pr\left[a \le |h_{sd}|^2 < b\right], \quad (15)$$

where

$$a = \frac{\tilde{\gamma}_0 - (l-k)P_r|h_{rd}|^2}{\sum_{m=1}^{l} P_{s,m}},\ b = \frac{\tilde{\gamma}_0 - (l-k-1)P_r|h_{rd}|^2}{\sum_{m=1}^{l-1} P_{s,m}}.$$

We observe that b should not be negative, i.e.,

$$b = \frac{\tilde{\gamma}_0 - (l-k-1)P_r|h_{rd}|^2}{\sum_{m=1}^{l-1} P_{s,m}} \ge 0,$$

which implies that $|h_{rd}|^2 \le \frac{\tilde{\gamma}_0}{(l-k-1)P_r}$. When $a < 0$, it means $|h_{rd}|^2 > \frac{\tilde{\gamma}_0}{(l-k)P_r}$. When $a \ge 0$, it means $|h_{rd}|^2 \le \frac{\tilde{\gamma}_0}{(l-k)P_r}$.

Therefore, the conditional probability (15) can be calculated as follows

$$q_{l,k} = \left[\int_{|h_{rd}|^2=0}^{\frac{\tilde{\gamma}_0}{(l-k)P_r}} \int_{|h_{sd}|^2=a}^{b} + \int_{|h_{rd}|^2=\frac{\tilde{\gamma}_0}{(l-k)P_r}}^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} \int_{|h_{sd}|^2=0}^{b} \right] \times \lambda_{sd} e^{-\lambda_{sd}|h_{sd}|^2} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathrm{d}|h_{sd}|^2 \mathrm{d}|h_{rd}|^2 \triangleq A_1 + A_2 - A_3, \quad (16)$$

where

$$A_1 = \int_0^{\frac{\tilde{\gamma}_0}{(l-k)P_r}} e^{-\lambda_{sd}a} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathrm{d}|h_{rd}|^2 = \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k)P_r}{\sum_{m=1}^{l} P_{s,m}} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l} P_{s,m}}} \right), \quad (17)$$

$$A_2 = \int_{\frac{\tilde{\gamma}_0}{(l-k)P_r}}^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathrm{d}|h_{rd}|^2 = e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}}, \quad (18)$$

$$A_3 = \int_0^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} e^{-\lambda_{sd}b} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathrm{d}|h_{rd}|^2 = \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k-1)P_r}{\sum_{m=1}^{l-1} P_{s,m}} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}}} \right). \quad (19)$$

In case of $k = l$, the conditional probability $q_{l,l}$ is given by

$$q_{l,l} = \Pr\left[\gamma_{d,l-1,l-1} < \gamma_0, \gamma_{d,l,l} \geq \gamma_0\right] = \Pr\left[\frac{\tilde{\gamma}_0}{\sum_{m=1}^{l} P_{s,m}} \leq |h_{sd}|^2 < \frac{\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}} \right] = e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l} P_{s,m}}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}}}. \quad (20)$$

When $l = L$, to calculate the conditional probability $q_{L,k}$, we also consider two scenarios: (i) $k = 1, 2, ..., L-1$, and (ii) $k = L$. For any $k = 1, 2, ..., L-1$, the conditional probability $q_{L,k}$ is given by

$$q_{L,k} = \Pr\left[\gamma_{d,L-1,k} < \gamma_0\right] = \Pr\left[|h_{sd}|^2 < \frac{\tilde{\gamma}_0 - (L-k-1)P_r|h_{rd}|^2}{\sum_{m=1}^{L-1} P_{s,m}} \right] \triangleq B_1 - B_2, \quad (21)$$

where

$$B_1 = 1 - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(L-k-1)P_r}}, \quad (22)$$

$$B_2 = \frac{\lambda_{rd}}{\frac{\lambda_{sd}(L-k-1)P_r}{\sum_{m=1}^{L-1} P_{s,m}} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(L-k-1)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{L-1} P_{s,m}}} \right). \quad (23)$$

For $k = L$, the conditional probability $q_{L,L}$ is given by

$$q_{L,L} = \Pr\left[\gamma_{d,L-1,L-1} < \gamma_0\right] = 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{L-1} P_{s,m}}}. \quad (24)$$

C. Average Total Transmission Power

Based on the analysis in the previous subsections, the average total transmission power in (9) for the cooperative H-ARQ relay protocol can be calculated as follows:

$$\bar{P}(L) = P_{s,1} e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1} \left(\sum_{k=1}^{l-1} C_{l,k} + C_{l,l} \right) + \left(\sum_{k=1}^{L-1} D_k + D_L \right), \quad (25)$$

where

$$C_{l,k} = q_{l,k} \Pr\left[T_r = k\right] R_{l,k} = (A_1 + A_2 - A_3) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k} P_{s,m}}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k-1} P_{s,m}}} \right) \times \left[\sum_{m=1}^{l} P_{s,m} + (l-k)P_r \right],$$

$$C_{l,l} = q_{l,l} \Pr\left[T_r = l\right] R_{l,l} = \left(e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l} P_{s,m}}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}}} \right) \times \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}}} \right) \sum_{m=1}^{l} P_{s,m},$$

$$D_k = q_{L,k} \Pr\left[T_r = k\right] R_{L,k} = (B_1 - B_2) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k} P_{s,m}}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{k-1} P_{s,m}}} \right) \times \left[\sum_{m=1}^{L} P_{s,m} + (L-k)P_r \right],$$

$$D_L = q_{L,L} \Pr\left[T_r = L\right] R_{L,L} = \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{L-1} P_{s,m}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{L-1} P_{s,m}}} \right) \sum_{m=1}^{L} P_{s,m},$$

in which A_1, A_2 and A_3 are specified in (17)–(19), and B_1 and B_2 are specified in (22) and (23), respectively. When $L = 2$, the average total transmission power expression $\bar{P}$ in (25) reduces to

$$\bar{P}(L=2) = P_{s,1} + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} \right) \left(P_{s,2} + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{P_{s,1}}} P_r \right). \quad (26)$$

The general closed-form expression in (25) for the average total transmission power is certainly complicated. Below, we are able to develop an asymptotically tight approximation. If we approximate e^{-x} by its Taylor expansion $1 - x + \frac{1}{2}x^2$ for small x (corresponding to high SNR), then the average total transmission power in (25) can be approximated at high SNR scenario as follows

$$\bar{P}(L) \approx P_{s,1} + \frac{\lambda_{sd}\tilde{\gamma}_0(P_{s,2} + P_r)}{P_{s,1}} + \sum_{l=3}^{L} \frac{\lambda_{sd}\tilde{\gamma}_0(P_{s,l} + P_r)}{\sum_{m=1}^{l-1} P_{s,m}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{l-2} P_{s,m}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r} \right). \quad (27)$$

The quantity of the (asymptotic) approximation of the average total transmission power in (27) is examined numerically and by simulations in the following section.

IV. Numerical and Simulation Results

In this section, we present numerical and simulation results to illustrate the average total power consumption for the cooperative H-ARQ relay scheme. In all studies, the variances of the channels h_{sd}, h_{sr} and h_{rd} are assumed to be $\frac{1}{\lambda_{sd}} = \frac{1}{\lambda_{sr}} = \frac{1}{\lambda_{rd}} = 1$. The targeted SNR is set at $\gamma_0 = 10$ dB. The source transmission power is assumed to be fixed over all transmission rounds, i.e. $P_{s,m} = P_s$ for $1 \leq m \leq L$, and equal to the relay power, i.e. $P_s = P_r = P$.

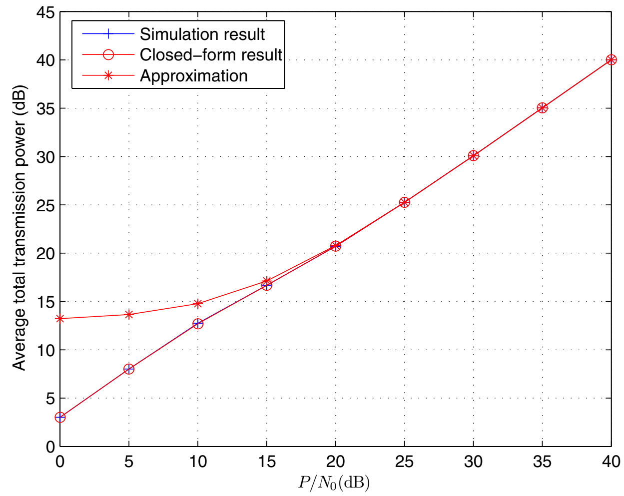

Fig. 2. Average total power consumption per information unit (packet) with equal source/relay power assignment $P_s = P_r = P$, $\gamma_0 = 10$ dB, $L = 2$.

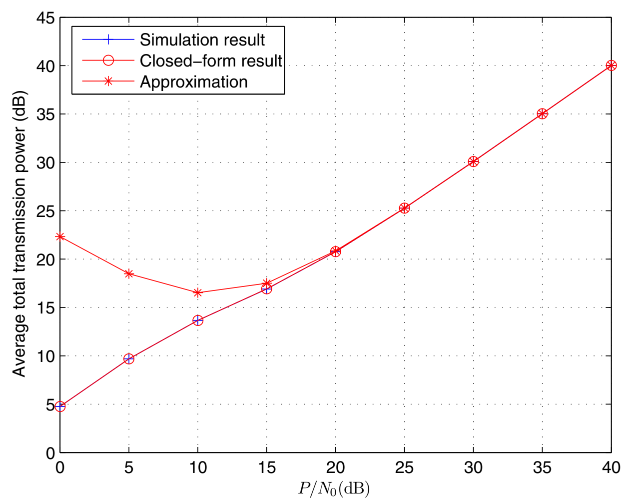

Fig. 3. Average total power consumption per information unit (packet) with equal source/relay power assignment $P_s = P_r = P$, $\gamma_0 = 10$ dB, $L = 3$.

Figs. 2, 3, and 4 show the average total transmission power of the cooperative H-ARQ protocol for $L = 2$, 3, and 4, respectively. In each figure, we compare the closed-form expression in (25) and the approximation result in (27) with the simulation result. Apparently, we can see that the closed-form result of the average total transmission power matches exactly with the simulation curve in each figure. We also observe that the approximation is loose at low SNR and tight at high SNR. The approximation curve is almost indistinguishable from the closed-form and simulated curves for SNR above 15 dB in each figure.

V. Conclusion

In this paper, we investigated for the first time the average total transmission power consumed by the cooperative H-ARQ protocol operating in a quasi-static Rayleigh fading environment. Specifically, we developed an analytical approach to

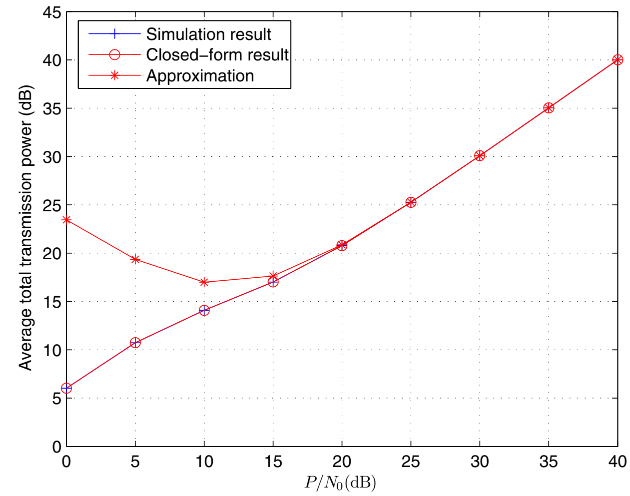

Fig. 4. Average total power consumption per information unit (packet) with equal source/relay power assignment $P_s = P_r = P$, $\gamma_0 = 10$ dB, $L = 4$.

obtain a closed-form expression of the average total transmission power. The closed-form result is valid for any number of maximum retransmission rounds L allowed in the protocol in which the source may use different transmission power levels in different rounds. Since the closed-form expression of the average total transmission power is complicated for large L, we also developed an approximation which is asymptotically tight at high SNR. Numerical and simulation results illustrate and validate our theoretical development.

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